Exploring Hyperobjects: Inside The Klein Bottle

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Abstract

The idea that 3-D projections of 4-D objects are analogous to 2-D projections of 3-D objects is an old one, but the ability to represent and explore these 3-D images in detail was impossible before the advent of computers and 3-D modeling programs. These 3-D objects are called "hyperobjects." A three dimensional Klein bottle can be considered a hyperobject, because there is an apparent self-intersection in 3-D that would not exist in 4-D. This paper describes a project in which a CAD model of a Klein bottle is created and then viewed from the inside.

1. The Simplified Klein Bottle

1.1 Introduction. I was working on a project creating 3-D models of various hyperobjects, assigning very reflective materials and exploring the inside of these objects with renderings and animations to produce some quite remarkable images. I though it would be interesting to see what a Klein bottle looked like from the inside, rendered like these other objects. The Klein bottle is a surface which has only one side and closes on itself having no sharp boundaries. Any two points on the surface can be connected by an unbroken line.

Construction of the Klein bottle is usually described as the process of deforming an ordinary bottle by bending the neck around, putting it through the side and connecting it to an opening in the bottom. I had no special interest in the topology of the surface, I simply needed a model of the bottle. The CAD program that I was using at the time did not have deformable modeling, so I was looking for an easy way to construct a complicated surface. The relationship of the Klein bottle to tori is well known, but lends itself more to geometric description of the surface than to actual construction of the object. I thought I could create a series of circles that could be lofted to create the surface, and played with this idea briefly. I soon realized that I would have to loft different parts of the surface and join them together. This led me to look for simple ways to create the individual parts.

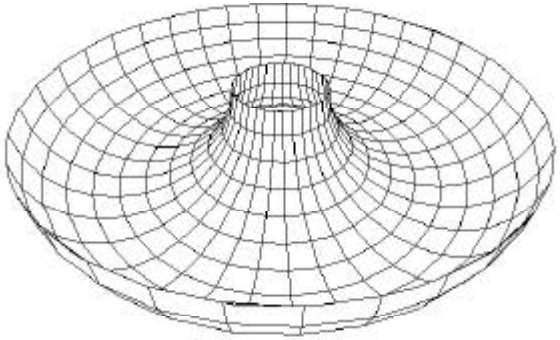

Figure 1: *The bottom half of the torus.*

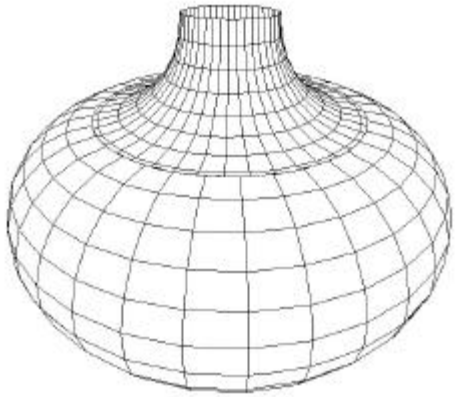

Figure 2: *The reassembled torus.*

I briefly considered inserting a cylinder into the base to make it taller, but then realized that I could change the tube diameter of the base torus to two units and make it work perfectly. (Figure 3) It is these two tori that make up what I have come to call "the simplified Klein bottle."

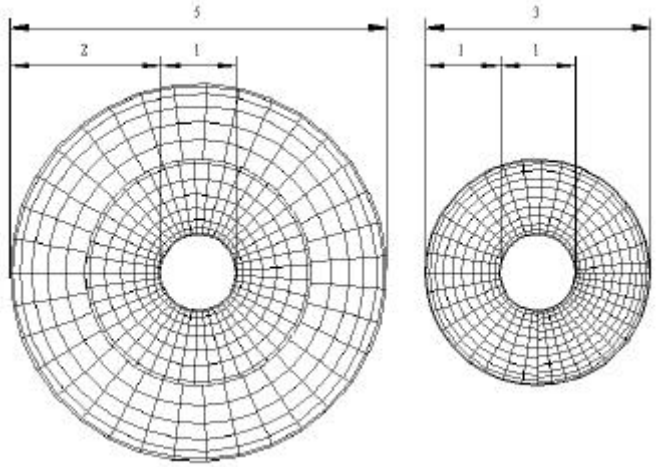

Figure 3: *The proportions of the two tori used to construct the simplified Klein bottle.*

In order to produce the model of the actual bottle, I cut a hole where the neck passes through the base. Other than this both tori are complete. This object is so elegant in its simplicity that I assumed it must be well known, yet I have been unable to find a published example nor anyone who is familiar with it. All common illustrations show elongated bottles bent around on themselves. I am not a mathematician, but it would seem to me that the simplified Klein bottle would also produce a simplified descriptive geometry. I won't claim to have invented it, because I don't believe that such things can be invented.

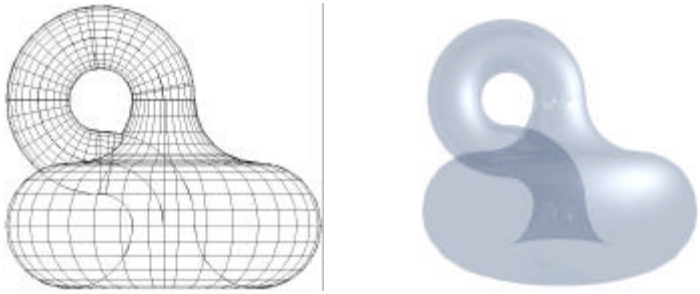

Figure 4: *An elevation and a rendering of the simplified Klein bottle.*

2. Image Generators

2.1 Hyperobjects. I discovered the simplified Klein bottle while working on the larger project of creating, rendering and animating hyperobjects. The idea that a cube-within-cube is a 3-D representation of a 4-D cube is well known. This representation is analogous to a square-within-square as a 2-D projection of a cube. (Figures 5 and 6.) I stretched this analogy to describe a whole class of hyperobjects which I created in virtual 3-D. They were an ontologically interesting metaphor of a fourth dimension, because even though they were solids and single objects they could be viewed from the inside. This property was also epistemologically essential to the analogy.

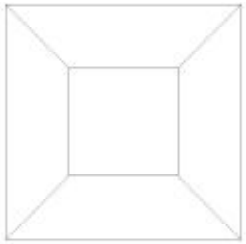

Figure 5: *A Cube in 2-D*

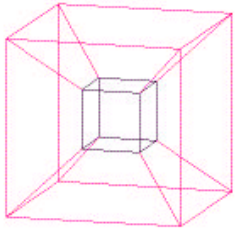

Figure 6: *A (2-D representation of) a Super-cube in 3-D*

These objects in and of themselves were of little interest. However, when created and rendered as reflective materials, they produced quite remarkable images, containing fractals, recursions, and chaotic structures. The rendered images are highly conceptual in that they are a result of the rendering process, which I could set in motion, but not control. Although many of these images could be found by manually placing the camera within the object, some of the most remarkable and unpredictable were frames extracted from animations. The effect of this was to remove my direct involvement one step further. (Figures 7 and 8)

2.2 Into the Klein Bottle. These hyperobjects are metaphorically different from the Klein bottle, in that they are solids; the Klein bottle is a surface. Nevertheless, the Klein bottle can be regarded as a type of hyperobject. As such I consider it part of the same series of image generators. The images are of greater visual interest than academic. So, without further ado, here are some images.

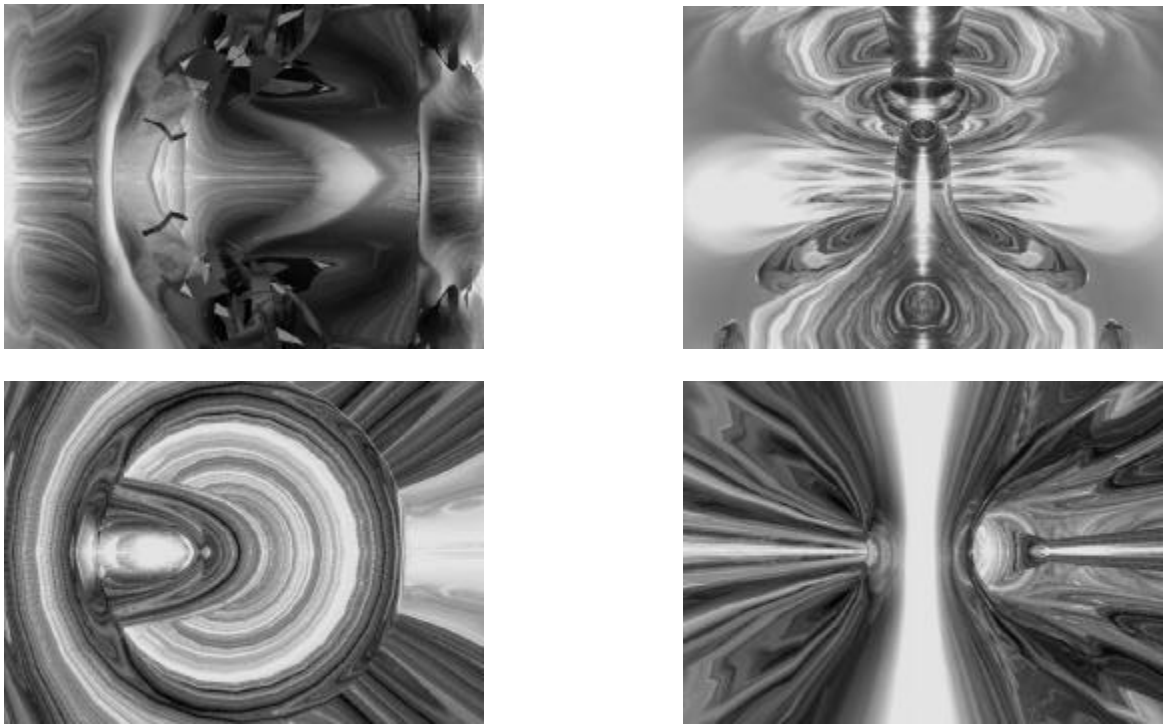

Figure 9: *Images from inside the Klein bottle.*

2.3 Images as Art. There is a great divide in the acceptance of electronic media in the production of art and the presentation of art. A fine art photograph, for example, can be computer manipulated to great extremes, but it will still need to be reproduced as a paper print. I don't mean to imply that conventional printing methods are superior to digital ones, but that prints of any form are much more acceptable as art than purely electronic images. I consider the images above part of a larger conceptual whole, and this will include presentation of images in a series of paper image-text prints. The real dilemma to the artist working in digital media lies in the presentation of virtual objects and animations that are best presented electronically. There are numerous ways that this can be done in a formal setting, such as a gallery, but how can the artist present these images electronically in a way distinctive from the plethora of ordinary images that bombard us daily.